First Order Linear Differential Systems with Singularities

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MATHIS - XLIM - Université de Limoges

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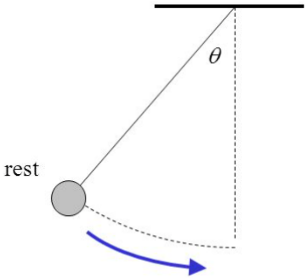
Workshop Doctorants XLIM 2017

- Phenomena of nature which involve movements can be mathematically modeled by differential equations.
- Often these equations are not linear.
  For example, the simple pendulum given by:

$$\theta''(t) = -\frac{g}{\ell}\sin(\theta(t)).$$



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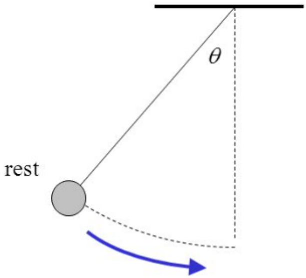

$\hookrightarrow$ For small variation, non-linear phenomena can be seen locally as linear.

Context

We treat linear systems of the form:

$$[A] : Y' = A(x)Y,$$

- A has coefficients in $\mathcal{C}((x))$: $A(x) = \frac{1}{x^{q+1}}(A_0 + A_1 x + \ldots)$.
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- Symbolic resolution.
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- Proceed efficiently and accurately.

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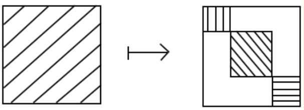
Strategy: get a refined structure

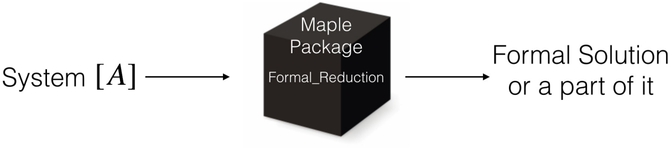
- First step: Maximal decomposition.



- Second step: each little block
  1. has one type of irregularity.
  2. we find theoretically the degree r of the field extension $\mathcal{C}((x^{1/r}))$.
- Third step: we continue using classical tools.

Results

- New algorithm for formal reduction with advantages:
  - Reduce computation costs by computing in the smallest necessary field extensions.
  - Certified computation.
- A maple package containing all functions of the algorithm.



Thank you!