> $restart:$

> $with(SplittingLemma):$

> $with(MiyakeReductionViaLP)$

$$[ExpPartsMiyakeReduction] \quad (1)$$

> $with(RankReductionViaLP):$

> $infolevel[MiyakeReductionViaLP] := 1:$

Exemple 1

> $A := \begin{bmatrix} 0 & 0 & x & 0 \\ 1 & -x^2 & x^2 & -x^2 \\ 0 & 1 & x^2 & 0 \\ x^2 & x^2 & 0 & -x^2 \end{bmatrix} : p := 4 :$

> $ExpPartsMiyakeReduction(A, 4, x, t)$

```
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-2 : irregular singular V
fractional
ExpPartsMiyakeReduction:  Volevic is fractional 1/3
ExpPartsMiyakeReduction: Computing Volevic numbers time: 0.7e-2
ExpPartsMiyakeReduction: Volevic Numbers [-2/3 -1/3 0 0]
ExpPartsMiyakeReduction: LeadingMatrice not invertible with V
fractional : we prepare A0
ExpPartsMiyakeReduction: Volevic  0.7e-2
ExpPartsMiyakeReduction: DetA0  0.
ExpPartsMiyakeReduction: NullSpace  0.4e-2
ExpPartsMiyakeReduction: Gauss elimination  0.1e-2
ExpPartsMiyakeReduction: Volevic  0.6e-2
ExpPartsMiyakeReduction: Miyake decomposition prepareA0 time:
0.29e-1
ExpPartsMiyakeReduction: We separate the nilpotent part and
recall for the two sub systmes
ExpPartsMiyakeReduction: SplittingTdevelop time:  .134
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-2 : irregular singular V
fractional
ExpPartsMiyakeReduction:  Volevic is fractional 1/3
ExpPartsMiyakeReduction: Computing Volevic numbers time: 0.7e-2
ExpPartsMiyakeReduction: Volevic Numbers [0 1/3 2/3]
ExpPartsMiyakeReduction: all eigenvalues are different from 0 [1
-1/2-((1/2)*I)*3^(1/2) -1/2+((1/2)*I)*3^(1/2)]
ExpPartsMiyakeReduction: We ramified with c*x^3
ExpPartsMiyakeReduction: Volevic after ramification 1
ExpPartsMiyakeReduction: Compute Volevic numbers time:  0.7e-2
ExpPartsMiyakeReduction: Computing Volevic Numbers [0 1 2]
ExpPartsMiyakeReduction: We flat the developpement
ExpPartsMiyakeReduction: Smart ramification x^3
ExpPartsMiyakeReduction:  with splitting the system after
ramification time:  0.80e-1
```

```
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = t^3
1/x^11+(1/3)/x^7-(1/3)/x^3] with ramification x = t^3
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = t
-1/x^2] with ramification x = t
```

$$\left[x=t^3, \frac{1}{x^{11}}+\frac{1}{3\,x^7}-\frac{1}{3\,x^3}\right], \left[x=t, -\frac{1}{x^2}\right] \qquad \textbf{(1.1)}$$

>

Exemple 2

> $A := \begin{bmatrix} x^5 & -x^3 & x^3-1 & -x^3 & -1 & -x^4 \\ 0 & 0 & x^2-x & x^6+x & x & -x \\ x^2 & x^6-x^5+x^2 & -x^3 & -x^3+1 & -x^4 & -x^5+x^3 \\ 0 & 0 & x-1 & x^6+x^4-x^3 & 1 & x^5 \\ 0 & x^2 & x^5 & -x^5 & -x^6+x^3 & -x^2 \\ -x^4 & x^6-x^4+x & -x^4 & 1 & -x^2 & x^2 \end{bmatrix}:$

> *ExpPartsMiyakeReduction*$(A, 7, x, t)$

```
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-1 : irregular singular V integer
ExpPartsMiyakeReduction: We flat the developpement
ExpPartsMiyakeReduction: case 2-1-b: for all roots we split
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = t
RootOf(_Z^2+1)/x^7-(1/2)*RootOf(_Z^2+1)/x^6+(3/8)*RootOf(_Z^2+1)
/x^5+(1/2)/x^5-(5/16)*RootOf(_Z^2+1)/x^4-(173/128)*RootOf
(_Z^2+1)/x^3-(399/256)*RootOf(_Z^2+1)/x^2+(183/1024)*RootOf
(_Z^2+1)/x-2/x] with ramification x = t
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-1 : irregular singular V integer
ExpPartsMiyakeReduction: We flat the developpement
ExpPartsMiyakeReduction: case 2-1-b: for all roots we split
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = t
RootOf(_Z^2+1)/x^6-RootOf(_Z^2+1)/x^5+1/x^5-2*RootOf(_Z^2+1)
/x^4+2/x^4-6*RootOf(_Z^2+1)/x^3+21/x^3+(55/2)*RootOf(_Z^2+1)
/x^2+131/x^2+(1129/2)*RootOf(_Z^2+1)/x+721/x] with ramification
x = t
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-2 : irregular singular V
fractional
ExpPartsMiyakeReduction:  Volevic is fractional 1/2
ExpPartsMiyakeReduction: Computing Volevic numbers time: 0.6e-2
ExpPartsMiyakeReduction: Volevic Numbers [-1/2 0]
ExpPartsMiyakeReduction: all eigenvalues are different from 0
[I*2^(1/2) -I*2^(1/2)]
ExpPartsMiyakeReduction: We ramified with c*x^2
```

```
ExpPartsMiyakeReduction: Volevic after ramification 1
ExpPartsMiyakeReduction: Compute Volevic numbers time:  0.5e-2
ExpPartsMiyakeReduction: Computing Volevic Numbers [0 1]
ExpPartsMiyakeReduction: We flat the developpement
ExpPartsMiyakeReduction: Smart ramification -2*x^2
ExpPartsMiyakeReduction:  with splitting the system after
ramification time:  0.66e-1
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = -2*t^2
-(1/32)/x^11+(1/32)/x^10+(3/32)/x^9-(5/32)/x^8-(25/64)/x^7+
(41/16)/x^6-(261/8)/x^4+(13057/256)/x^3+(2875/8)/x^2] with
ramification x = -2*t^2
```

$$\left[x=t, \frac{RootOf(\_Z^2+1)}{x^7} - \frac{1}{2}\,\frac{RootOf(\_Z^2+1)}{x^6} + \frac{3}{8}\,\frac{RootOf(\_Z^2+1)}{x^5} + \frac{1}{2\,x^5} - \frac{5}{16}\,\frac{RootOf(\_Z^2+1)}{x^4} - \frac{173}{128}\,\frac{RootOf(\_Z^2+1)}{x^3} - \frac{399}{256}\,\frac{RootOf(\_Z^2+1)}{x^2} + \frac{183}{1024}\,\frac{RootOf(\_Z^2+1)}{x} - \frac{2}{x}\right], \left[x=t, \frac{RootOf(\_Z^2+1)}{x^6} - \frac{RootOf(\_Z^2+1)}{x^5} + \frac{1}{x^5} - \frac{2\,RootOf(\_Z^2+1)}{x^4} + \frac{2}{x^4} - \frac{6\,RootOf(\_Z^2+1)}{x^3} + \frac{21}{x^3} + \frac{55}{2}\,\frac{RootOf(\_Z^2+1)}{x^2} + \frac{131}{x^2} + \frac{1129}{2}\,\frac{RootOf(\_Z^2+1)}{x} + \frac{721}{x}\right], \left[x = -2\,t^2, -\frac{1}{32\,x^{11}} + \frac{1}{32\,x^{10}} + \frac{3}{32\,x^9} - \frac{5}{32\,x^8} - \frac{25}{64\,x^7} + \frac{41}{16\,x^6} - \frac{261}{8\,x^4} + \frac{13057}{256\,x^3} + \frac{2875}{8\,x^2}\right] \quad (2.1)$$

Exemple 3

> $A := [[4\,x^3, 0, 1, 0, 0, -x^6 - 5\,x^4, 0, 0],$
> $[-5\,x^4, -9\,x^5, 6\,x^2, 1 - 6\,x, -2\,x^2, 0, 0, 0],$
> $[0, 1, -10\,x^2, -x^6 - 5\,x^5 + x^4, -2\,x^3, 10\,x, 0, 6\,x^5],$
> $[0, 0, 2\,x^6, -x^2, 0, 0, -8\,x^3, -5\,x^4],$
> $[-7\,x^3, -3\,x^4, 0, -4\,x^4 - 6\,x^2, 0, -6\,x^3, 0, 8\,x],$
> $[-x^6 - 9\,x^5 + 1, 0, 0, 6\,x^6, -10\,x^4, 8\,x^3 + 1, 0, -x^6 + 8\,x^3],$
> $[2\,x^6, 0, 0, x^3 + 1, 0, 7\,x^4, 0, -8\,x^2],$
> $[-5\,x^5, 0, 0, 0, 0, -6\,x^4 + 10\,x^2, 0, 3\,x^4 - 10\,x]]:$

>

> $ExpPartsMiyakeReduction(A, 7, x, t)$

```
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-1 : irregular singular V integer
ExpPartsMiyakeReduction: We flat the developpement
ExpPartsMiyakeReduction: case 2-1-b: for all roots we split
```

```
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = t
1/x^7+10/x^6-200/x^5+6968/x^4-298725/x^3+14242192/x^2
-725038149/x] with ramification x = t
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-2 : irregular singular V
fractional
ExpPartsMiyakeReduction:  Volevic is fractional 1/2
ExpPartsMiyakeReduction: Computing Volevic numbers time: 0.26e-1
ExpPartsMiyakeReduction: Volevic Numbers [-3/2 -1 -1/2 -1/2 0 0
0]
ExpPartsMiyakeReduction: LeadingMatrice not invertible with V
fractional : we prepare A0
ExpPartsMiyakeReduction: Volevic  0.27e-1
ExpPartsMiyakeReduction: DetA0  0.1e-2
ExpPartsMiyakeReduction: NullSpace  0.3e-2
ExpPartsMiyakeReduction: Gauss elimination  0.1e-2
ExpPartsMiyakeReduction: Volevic  0.25e-1
ExpPartsMiyakeReduction: DetA0  0.
ExpPartsMiyakeReduction: NullSpace  0.4e-2
ExpPartsMiyakeReduction: Gauss elimination  0.1e-2
ExpPartsMiyakeReduction: Volevic  0.22e-1
ExpPartsMiyakeReduction: DetA0  0.
ExpPartsMiyakeReduction: NullSpace  0.2e-2
ExpPartsMiyakeReduction: Gauss elimination  0.1e-2
ExpPartsMiyakeReduction: Volevic  0.27e-1
ExpPartsMiyakeReduction: DetA0  0.1e-2
ExpPartsMiyakeReduction: NullSpace  0.2e-2
ExpPartsMiyakeReduction: Gauss elimination  0.1e-2
ExpPartsMiyakeReduction: Volevic  0.23e-1
ExpPartsMiyakeReduction: DetA0  0.
ExpPartsMiyakeReduction: NullSpace  0.3e-2
ExpPartsMiyakeReduction: Gauss elimination  0.1e-2
ExpPartsMiyakeReduction: Volevic  0.24e-1
ExpPartsMiyakeReduction: Miyake decomposition prepareA0 time:
.296
ExpPartsMiyakeReduction: We separate the nilpotent part and
recall for the two sub systmes
ExpPartsMiyakeReduction: SplittingTdevelop time:  2.515
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-2 : irregular singular V
fractional
ExpPartsMiyakeReduction:  Volevic is fractional 1/2
ExpPartsMiyakeReduction: Computing Volevic numbers time: 0.7e-2
ExpPartsMiyakeReduction: Volevic Numbers [0 1/2]
ExpPartsMiyakeReduction: all eigenvalues are different from 0
[I*10^(1/2) -I*10^(1/2)]
ExpPartsMiyakeReduction: We ramified with c*x^2
ExpPartsMiyakeReduction: Volevic after ramification 1
ExpPartsMiyakeReduction: Compute Volevic numbers time:  0.7e-2
ExpPartsMiyakeReduction: Computing Volevic Numbers [0 1]
ExpPartsMiyakeReduction: We flat the developpement
ExpPartsMiyakeReduction: Smart ramification -10*x^2
ExpPartsMiyakeReduction:  with splitting the system after
ramification time:  0.90e-1
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = -10*t^2
(1/1000000)/x^13-(1/200000)/x^12+(131/2000000)/x^11-(19/20000)
/x^10+(143239/8000000)/x^9-(69539/200000)/x^8+
(113459131/16000000)/x^7-(466768/3125)/x^6+
```

```
(82655665103/25600000)/x^5-(3560584523/50000)/x^4+
(408609419976357/256000000)/x^3-(1812600809291/50000)/x^2] with
ramification x = -10*t^2
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-1 : irregular singular V integer
ExpPartsMiyakeReduction: We flat the developpement
ExpPartsMiyakeReduction: case 2-1-b: for all roots we split
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = t
-10/x^6+(4/25)/x^5-(17311/12500)/x^4+(687971/3125000)/x^3+
(12315211171/78125000)/x^2-(843753113492397/390625000000)/x]
with ramification x = t
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: Reduce the Poincare rank
ExpPartsMiyakeReduction: case 2-2 : irregular singular V
fractional
ExpPartsMiyakeReduction:  Volevic is fractional 1/2
ExpPartsMiyakeReduction: Computing Volevic numbers time: 0.11e-1
ExpPartsMiyakeReduction: Volevic Numbers [-1/2 0 1/2 1]
ExpPartsMiyakeReduction: all eigenvalues are different from 0 [
(2*I)*2^(1/2) -(2*I)*2^(1/2) (2/5)*10^(1/2) -(2/5)*10^(1/2)]
ExpPartsMiyakeReduction: We ramified with c*x^2
ExpPartsMiyakeReduction: Volevic after ramification 1
ExpPartsMiyakeReduction: Compute Volevic numbers time:  0.12e-1
ExpPartsMiyakeReduction: Computing Volevic Numbers [0 1 2 3]
ExpPartsMiyakeReduction: We flat the developpement
ExpPartsMiyakeReduction: Smart ramification -8*x^2
ExpPartsMiyakeReduction:  with splitting the system after
ramification time:  .174
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = -8*t^2
-(1/32768)/x^11+(1/196608)/x^10-(29/1769472)/x^9-
(66137/127401984)/x^8-(8799649/22932357120)/x^7-
(556575917/412782428160)/x^6+(27825936553/4953389137920)/x^5-
(2519093693257/222902511206400)/x^4-
(970668079239 0541/32097961613721600)/x^3-
(4900145795585 6431/866644963570483200)/x^2] with ramification x
= -8*t^2
ExpPartsMiyakeReduction: Smart ramification (8/5)*x^2
ExpPartsMiyakeReduction:  with splitting the system after
ramification time:  .171
ExpPartsMiyakeReduction: a call of the algorithme
ExpPartsMiyakeReduction: we get an exponential part [x = (8/5)*
t^2 (3125/32768)/x^11-(3875/98304)/x^10+(528425/7077888)/x^9+
(171923293/637009920)/x^8-(254308345693/1146617856000)/x^7-
(161270312110441/257989017600000)/x^6+
(59660172017758747/15479341056000000)/x^5-
(12990039338482186199/348285173760000000)/x^4+
(1421410099469277912836123/250765325107200000000)/x^3-
(213078798250654103919264991/4231664861184000000000)/x^2]
with ramification x = (8/5)*t^2
```

$$\left[x=t, \frac{1}{x^7}+\frac{10}{x^6}-\frac{200}{x^5}+\frac{6968}{x^4}-\frac{298725}{x^3}+\frac{14242192}{x^2}-\frac{725038149}{x}\right], \left[x= -10\,t^2, \frac{1}{1000000\,x^{13}}-\frac{1}{200000\,x^{12}}+\frac{131}{2000000\,x^{11}}-\frac{19}{20000\,x^{10}}+\frac{143239}{8000000\,x^9} -\frac{69539}{200000\,x^8}+\frac{113459131}{16000000\,x^7}-\frac{466768}{3125\,x^6}+\frac{82655665103}{25600000\,x^5}-\frac{3560584523}{50000\,x^4}\right. \quad \textbf{(3.1)}$$

$$+\frac{408609419976357}{256000000\,x^{3}}-\frac{1812600809291}{50000\,x^{2}}\Bigg],\Bigg[x=t,\,-\frac{10}{x^{6}}+\frac{4}{25\,x^{5}}-\frac{17311}{12500\,x^{4}}$$

$$+\frac{687971}{3125000\,x^{3}}+\frac{12315211171}{78125000\,x^{2}}-\frac{843753113492397}{390625000000\,x}\Bigg],\Bigg[x=-8\,t^{2},\,-\frac{1}{32768\,x^{11}}$$

$$+\frac{1}{196608\,x^{10}}-\frac{29}{1769472\,x^{9}}-\frac{66137}{127401984\,x^{8}}-\frac{8799649}{22932357120\,x^{7}}$$

$$-\frac{556575917}{412782428160\,x^{6}}+\frac{27825936553}{4953389137920\,x^{5}}-\frac{2519093693257}{222902511206400\,x^{4}}$$

$$-\frac{9706680792390541}{32097961613721600\,x^{3}}-\frac{49001457955856431}{866644963570483200\,x^{2}}\Bigg],\Bigg[x=\frac{8}{5}\,t^{2},\,\frac{3125}{32768\,x^{11}}$$

$$-\frac{3875}{98304\,x^{10}}+\frac{528425}{7077888\,x^{9}}+\frac{171923293}{637009920\,x^{8}}-\frac{254308345693}{1146617856000\,x^{7}}$$

$$-\frac{161270312110441}{257989017600000\,x^{6}}+\frac{59660172017758747}{154793410560000000\,x^{5}}-\frac{12990039338482186199}{3482851737600000000\,x^{4}}$$

$$+\frac{14214100994692779128361 23}{2507653251072000000000\,x^{3}}-\frac{21307879825065410391926 4991}{4231664861184000000000000\,x^{2}}\Bigg]$$

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