```
> restart :
> with(RankReductionViaLP);
```

[EchelonneThePrincipalMatrixFunction, MinimalNormalizedPath, MinimalNormalizedPathLenght, PrincipalMatrix, PrincipalMatrixFunction, RankReductionLP, VolevicNumbers, VolevicWeightWithLinearProgram] **(1)**

```
> infolevel[RankReductionViaLP] := 1 :
>
>
```

I-Réduction du rang de poincaré en utilisant un programme linéaire.

Exemple 1 : Reduce the Poincaré rank from 4 to 0

$$> RankReductionLP\left(\begin{bmatrix} -x^4+x^3 & 0 & x^5 \\ 2x^4-x^2+1 & 0 & 2x^6+4x^3+x^2 \\ x^3-x & 0 & -3x^4-x^3 \end{bmatrix}, 5, x\right)$$

```
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 3 [0 -3 -2]
RankReductionLP: Preparation of A0x
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 4 [-4 0 0]
RankReductionLP: Singular regular. Volevic is greater then p
```

$$\begin{bmatrix} -x & 2x^6+4x^3+x^2 & 2x^4-x^2+1 \\ 0 & 2x^3+1 & 2x \\ 1 & -2x^5-4x^2 & -2x^3+x-1 \end{bmatrix}, 1 \qquad \textbf{(1.1)}$$

Exemple 2

$$> RankReductionLP\left(\begin{bmatrix} -5x^8+x^3 & 0 & x^5 \\ 1 & 0 & 2x^{10}+4x^7+x^2 \\ -x & 0 & -3x^8-x^3 \end{bmatrix}, 9, x\right)$$

```
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 3 [0 -3 -2]
RankReductionLP: Preparation of A0x
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 11/2 [-11/2 0 0]
RankReductionLP: Singular irregular. Katz: 5/2 2.500000000
```

$$\left[\begin{array}{ccc} -2x^6+x^3 & 2x^{10}+4x^7+x^2 & 1 \\ -2x^2 & 2x^6+x^3 & 0 \\ 2x^4-4x & -2x^8-4x^5 & -5x^3 \end{array}\right], 4 \qquad \textbf{(2.1)}$$

Exemple 3

$$> RankReductionLP\left(\left[\begin{array}{ccc} -5x^{80}+x^{75} & 0 & x^{130} \\ 2x^{61}-x^{10}-4x^5+1 & -19x^{80} & 2x^{116}+57x^{60}+x^{55} \\ x^{30}+4x^{25}-x^{20} & 0 & -56x^{80}-x^{75} \end{array}\right], 81, x\right)$$

```
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 75 [0 -75 -55]
RankReductionLP: Preparation of A0x
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 80 [-80 0 0]
RankReductionLP: Singular regular. Volevic is greater then p
```

$$\left[\begin{array}{ccc} -x^5 & 2x^{116}+57x^{60}+x^{55} & 2x^{61}-x^{10}-4x^5+1 \\ 0 & 2x^{56}+1 & 2x \\ 1 & -2x^{111}-57x^{55} & -2x^{56}+x^5-1 \end{array}\right], 1 \qquad \textbf{(3.1)}$$

Exemple 4

$$> A := \left[\left[-\frac{31}{3}x^{50}+x^{49}, \frac{1}{3}x^{60}-\frac{1}{3}x^{59}, \frac{1}{3}x^{69}, -\frac{1}{3}x^{78}, x^{87}, x^{100}-x^{97}\right],\right.$$
$$\left[-x^{37}, -\frac{59}{3}x^{50}+x^{49}, 0, \frac{1}{3}x^{68}, 0, x^{87}\right],$$
$$\left[-\frac{1}{3}x^{28}, x^{40}+x^{37}, -\frac{89}{3}x^{50}+x^{49}, \frac{1}{3}x^{58}, \frac{1}{3}x^{68}, x^{77}\right],$$
$$\left[-\frac{1}{3}x^{19}-x^{17}, x^{28}, 0, -39x^{50}+x^{49}, x^{57}, \frac{1}{3}x^{69}\right],$$
$$\left[-\frac{4}{3}x^{10}+\frac{1}{3}x^{9}, \frac{1}{3}x^{20}-x^{18}, \frac{1}{3}x^{29}+x^{28}, x^{40}-\frac{1}{3}x^{38}, -49x^{50}+x^{49}+x^{47}, x^{60}\right.$$
$$\left.-x^{57}\right],$$
$$\left.\left[-1, \frac{1}{3}x^{10}, \frac{1}{3}x^{19}, -\frac{1}{3}x^{28}+x^{27}, x^{37}, -\frac{175}{3}x^{50}+x^{49}-x^{47}\right]\right]:$$

$> RankReductionLP(A, 51, x)$

```
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 47 [0 -10 -20 -30
-40 -50]
RankReductionLP: Preparation of A0x
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 142/3 [-51 -122/3
```

```
-91/3 -62/3 -31/3 0]
RankReductionLP: Singular irregular. Katz: 8/3 2.666666667
```

$$\left[\left[-\frac{29}{3}x^3+\frac{2}{3}x^2, -\frac{4}{3}x^3-\frac{1}{3}x^2+1, -\frac{1}{3}x+1, \frac{1}{3}x^2, \frac{1}{3}x^3, -x^3\right],\right.$$
$$\left[\frac{1}{3}x^2, -\frac{28}{3}x^3+\frac{4}{3}x^2, x^3-1, x, -x, -\frac{1}{3}x^3+\frac{1}{3}x^2\right],$$
$$\left[-x, -\frac{1}{3}x^2-x, -9x^3+x^2, 0, x, -\frac{1}{3}x^2-1\right],$$
$$\left[-x^3, -2x^3, x^3, -\frac{29}{3}x^3+\frac{4}{3}x^2+x, \frac{4}{3}x^3+1-x, -\frac{1}{3}x-\frac{4}{3}x^3+\frac{1}{3}x^2\right],$$
$$\left[0, -x^3, x^3, \frac{1}{3}x^2+x, -\frac{28}{3}x^3+x^2-x, -1-\frac{4}{3}x^3+\frac{1}{3}x^2\right],$$
$$\left.\left[0, 0, -x^3, -x, -\frac{1}{3}x^2+x, -9x^3+\frac{2}{3}x^2\right]\right], 4 \quad \textbf{(4.1)}$$

\>

II-Sous-procèdure:

a) Calcul du poids et des nombres de Volevic

Exemple 5

$$> A := \begin{bmatrix} 3x^6+4x^3-3x^2 & 4x^4+5x^3-3x^2-x & -1 \\ -4x^3+3x^2 & -4x^4-5x^3+3x^2+x & -5x^7+3x^6+1 \\ -3x^7-4x^6+5x^4-3x^3 & 3x^6+3x^5+7x^4-3x^3-x^2 & 8x^5+x^3-x \end{bmatrix}:$$

$p := 5:$

\> $R := map(ldegree, A, x);$

$$R := \begin{bmatrix} 2 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \quad \textbf{(5.1)}$$

\> $M := MinimalNormalizedPath(R)$

$$M := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \textbf{(5.2)}$$

\> $V := MinimalNormalizedPathLenght(R, M);$

$$V := 1 \quad \textbf{(5.3)}$$

> $s := VolevicNumbers(R, V)$

$$s := [-1, -1, 0] \quad (5.4)$$

Exemple 6

> $A := \begin{bmatrix} 0 & x^2 & 1 \\ 0 & 0 & 0 \\ 0 & x & 0 \end{bmatrix} : p := 5 :$

> $R := map(ldegree, A, x);$

$$R := \begin{bmatrix} \infty & 2 & 0 \\ \infty & \infty & \infty \\ \infty & 1 & \infty \end{bmatrix} \quad (6.1)$$

> $M := MinimalNormalizedPath(R)$

$$M := \infty \quad (6.2)$$

> $V := MinimalNormalizedPathLenght(R, M);$

$$V := \infty \quad (6.3)$$

> $s := VolevicNumbers(R, V)$

$$s := [0, 0, 0] \quad (6.4)$$

>

Exemple 7

> $A := \begin{bmatrix} x & x & x^3 \\ 2 & x^4 & 0 \\ x^2 & 3 & x^5 \end{bmatrix} : p := 3 :$

> $R := map(ldegree, A, x);$

$$R := \begin{bmatrix} 1 & 1 & 3 \\ 0 & 4 & \infty \\ 2 & 0 & 5 \end{bmatrix} \quad (7.1)$$

> $M := MinimalNormalizedPath(R)$

$$M := \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (7.2)$$

> $V := MinimalNormalizedPathLenght(R, M);$

$$V := \frac{1}{2} \quad (7.3)$$

> $s := VolevicNumbers(R, V)$

$$s := \left[0, -\frac{1}{2}, -1\right] \quad (7.4)$$

>

b) Matrice principale et matrice des termes principaux : A0, A0x

Exemple 5 suite

> $A0 := PrincipalMatrix(A, x, s, V);$

$$A0 := \begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \quad (8.1)$$

> $A0x := PrincipalMatrixFunction(A0, x, s, V);$

$$A0x := \begin{bmatrix} 0 & -x & -1 \\ 0 & x & 1 \\ 0 & -x^2 & -x \end{bmatrix} \quad (8.2)$$

c) Préparation de A0 et A0x quand A0 est nilpotent (augmenter le poids de Volevic)

Exemple 5 suite

> $A0, A0x;$

$$\begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & -x & -1 \\ 0 & x & 1 \\ 0 & -x^2 & -x \end{bmatrix} \quad (9.1)$$

>

> $EchelonneThePrincipalMatrixFunction(A, p, A0, A0x, s)$

(9.2)

$$\left[\begin{array}{ccc}1 & 0 & 0\\ -1 & 1 & 0\\ 1 & 0 & 1\end{array}\right], \left[\begin{array}{ccc}0 & -1 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{array}\right], \left[\begin{array}{ccc}0 & -x & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{array}\right], \left[\begin{array}{ccc}1 & 0 & 0\\ -1 & 1 & 0\\ x & 0 & 1\end{array}\right] \quad (9.2)$$

\>

\> $\left[\begin{array}{ccc}1 & 0 & 0\\ -1 & 1 & 0\\ x & 0 & 1\end{array}\right]^{-1} .A0x. \left[\begin{array}{ccc}1 & 0 & 0\\ -1 & 1 & 0\\ x & 0 & 1\end{array}\right]$

$$\left[\begin{array}{ccc}0 & -x & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{array}\right] \quad (9.3)$$

\>

Suite reduction du rang exemple 5

\> $A := \left[\begin{array}{ccc}3x^6 + 4x^3 - 3x^2 & 4x^4 + 5x^3 - 3x^2 - x & -1\\ -4x^3 + 3x^2 & -4x^4 - 5x^3 + 3x^2 + x & -5x^7 + 3x^6 + 1\\ -3x^7 - 4x^6 + 5x^4 - 3x^3 & 3x^6 + 3x^5 + 7x^4 - 3x^3 - x^2 & 8x^5 + x^3 - x\end{array}\right]:$

$p := 5:$

\> $RankReductionLP(A, 5, x)$

```
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 1 [-1 -1 0]
RankReductionLP: Preparation of A0x
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 3 [-3 0 0]
RankReductionLP: Preparation of A0x
RankReductionLP: Call of the algorithm
RankReductionLP: Volevic weight and numbers : 10/3 [-2/3 -10/3
0]
RankReductionLP: Singular irregular. Katz: 2/3 .6666666667
```

$$\left[\begin{array}{ccc}0 & 5x^4 - 3x^3 - 5x^2 + 3x + 3 & -5x^4 + 3x^3\\ 4x^4 + 5x^3 - 3x^2 - x & 3x^3 - x & -1\\ 4x^4 + 8x^3 - 4x^2 + x & 3x^3 - 8x^2 - 7x & 8x^2\end{array}\right], 2 \quad (10.1)$$

\>

\>