```
> restart :
> with(SmithNormalForm);
```

[CharacPoly, NullSpaceSeries, RationalKernelDecomposition, SmithNormalFormSeries, getseries] **(1)**

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>
```

Exemple 1: Forme normale de Smith approchée

$$> A := \begin{bmatrix} 0 & 6x & \frac{7}{x^3} - \frac{10}{x^2} & 0 \\ \frac{5}{x} & 0 & 10 & -x \\ -\frac{6}{x} & -\frac{3}{x^2} - \frac{8}{x} & -\frac{1}{x^3} & \frac{4}{x^3} - \frac{1}{x^2} - 2x \\ 0 & 0 & -\frac{2}{x} & 7 \end{bmatrix} :$$

```
> U, invU, M, V, invV := SmithNormalFormSeries(A, x, 1 , 2);
```

$$U, invU, M, V, invV := \begin{bmatrix} \frac{1}{7} + \frac{10}{49}x + \frac{100}{343}x^2 & 0 & 0 & 0 \\ \frac{1}{28} + \frac{47}{784}x + \frac{1929}{21952}x^2 & 0 & \frac{1}{4} + \frac{1}{16}x + \frac{1}{64}x^2 & 0 \\ 0 & \frac{1}{5} & 0 & 0 \\ \frac{8}{147}x^2 & 0 & 0 & \frac{4}{21} \end{bmatrix}, \quad \textbf{(1.1)}$$

$$\begin{bmatrix} 7-10x & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 1 & 4-x & 0 & 0 \\ 2x^2 & 0 & 0 & \frac{21}{4} \end{bmatrix}, \begin{bmatrix} \frac{1}{x^3} & 0 & 0 & 0 \\ 0 & \frac{1}{x^3} & 0 & 0 \\ 0 & 0 & \frac{1}{x} & 0 \\ 0 & 0 & 0 & x \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{3}{2}x^2 & \frac{3}{4}x + \frac{35}{16}x^2 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{3}{2}x^2 & \frac{3}{4}x + \frac{35}{16}x^2 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

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Exemple 2: Base d'un noyau approchée

$$> A := \left[\left[-28x^2 - 28x - 70 + \frac{1337}{8}x^3, -\frac{7}{x} + 56 - \frac{469}{4}x + \frac{3115}{8}x^2 - \frac{104321}{64}x^3, -70 + 7x - \frac{77}{4}x^2 + \frac{371}{4}x^3\right], \left[-14x^3 - 7x^2, 63x^2 - 14x - 133 - \frac{567}{2}x^3, 56x^3 - 7x^2\right], \left[-70 + 7x - \frac{105}{4}x^2 + \frac{623}{4}x^3, -\frac{7}{x} - 7 - \frac{413}{4}x + \frac{3787}{8}x^2 - \frac{129129}{64}x^3, -35x^2 + 42x - 70 + \frac{931}{8}x^3\right]\right]:$$

\>

$> U, invU, M, V, invV := SmithNormalFormSeries(A, x, 3, 3);$

$U, invU, M, V,$ **(2.1)**

$$invV := \left[\left[-\frac{1}{7} - \frac{8}{7}x - \frac{27}{4}x^2 - \frac{2397}{56}x^3, 0, 0\right], \left[-\frac{1}{70}x + \frac{3}{532}x^2 + \frac{3}{760}x^3, \frac{1}{1330} - \frac{17}{2660}x + \frac{3}{190}x^2 + \frac{2057}{10640}x^3, 0\right], \left[-1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{4}x^3, -\frac{1}{2} + \frac{3}{8}x - \frac{1}{8}x^2 - \frac{581}{32}x^3, 1\right]\right],$$

$$\begin{bmatrix} -7 + 56x - \frac{469}{4}x^2 + \frac{3115}{8}x^3 & 0 & 0 \\ -63x^3 + 14x^2 + 133x & 1330 + 11305x + \frac{136325}{2}x^2 & 0 \\ 7 + 7x + \frac{413}{4}x^2 - \frac{3787}{8}x^3 & -\frac{240065}{8}x^2 - \frac{20615}{4}x - 665 & 1 \end{bmatrix},$$

$$\left[\begin{array}{ccc}\frac{1}{x} & 0 & 0\\ 0 & x & 0\\ 0 & 0 & 0\end{array}\right], \left[\begin{array}{ccc}0 & 1 & -1+\frac{1}{2}x-\frac{1}{8}x^2-\frac{393}{16}x^3\\ 1 & -10x-84x^2-\frac{1017}{2}x^3 & \frac{1}{2}x^3\\ 0 & 0 & 1\end{array}\right],$$

$$\left[\begin{array}{ccc}-10x-84x^2-\frac{1017}{2}x^3 & 1 & -10x-79x^2-\frac{1869}{4}x^3\\ 1 & 0 & -1+\frac{1}{2}x-\frac{1}{8}x^2-\frac{393}{16}x^3\\ 0 & 0 & 1\end{array}\right]$$

\>

(2.2)

\> $CharacPoly(A, x)$

$$\lambda\,(\lambda+140)\,(\lambda+133) \qquad (2.3)$$

\> $p :=$ lambda :

\> $NullSpaceSeries(A, x,$ lambda$, p, 3, 3)$

$$\left[\left[\begin{array}{c}-1+\frac{1}{2}x-\frac{1}{8}x^2-\frac{393}{16}x^3\\ \frac{1}{2}x^3\\ 1\end{array}\right], 1\right] \qquad (2.4)$$

\>

Exemple 3: Decomposition du noyau

\> $A := \Big[\Big[-9-28x-28x^2+\frac{1337}{8}x^3, -\frac{7}{x}+56-\frac{469}{4}x+\frac{3115}{8}x^2-\frac{104321}{64}x^3, -70+7x-\frac{77}{4}x^2+\frac{371}{4}x^3\Big], \Big[-14x^3-7x^2, -72-14x+63x^2-\frac{567}{2}x^3, 56x^3-7x^2\Big], \Big[-70+7x-\frac{105}{4}x^2+\frac{623}{4}x^3, -\frac{7}{x}-7-\frac{413}{4}x+\frac{3787}{8}x^2-\frac{129129}{64}x^3, -9+42x-35x^2+\frac{931}{8}x^3\Big]\Big] : p := CharacPoly(A, x);$

$$p := (\lambda+72)\,(\lambda-61)\,(\lambda+79) \qquad (3.1)$$

\> $P, List := RationalKernelDecomposition(A, x,$ lambda$, p, 3, 3)$

$$P, List := \begin{bmatrix} -1+\frac{1}{2}x-\frac{1}{8}x^2-\frac{393}{16}x^3 & 1+\frac{1}{2}x-\frac{7}{8}x^2+\frac{3}{4}x^3 & x^2+x+1 \\ \frac{1}{2}x^3 & 2x^2-\frac{3}{2}x^3 & -x^3-2x^2-x \\ 1 & 1 & 1 \end{bmatrix},\ [1, 1, 1] \quad \textbf{(3.2)}$$

> $test := map\left(series, P^{-1}.A.P, x, 3\right)$

$$test := \begin{bmatrix} 61+O(x^3) & O(x^3) & O(x^3) \\ O(x^2) & -79+O(x^2) & O(x^2) \\ O(x^2) & O(x^2) & -72+O(x^2) \end{bmatrix} \quad \textbf{(3.3)}$$