```
> restart :
> infolevel[BarkatouReduction] := 1 :
> with(SplittingLemma) : with(BarkatouReduction) :
>
```

Exemple 1

$$> A := \frac{1}{x^4} \cdot \begin{bmatrix} 0 & 0 & x & 0 \\ 1 & -x^2 & x^2 & -x^2 \\ 0 & 1 & x^2 & 0 \\ x^2 & x^2 & 0 & -x^2 \end{bmatrix} :$$

```
> ExpPartsBarkatouReduction(A, x, t)
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: Apply Moser
ExpPartsBarkatouReduction: The leading Matrix
ExpPartsBarkatouReduction: case 1: Nilpotente and moser
irreducible: compute Katz
ExpPartsBarkatouReduction: Time for computing Katz 0.45e-1
ExpPartsBarkatouReduction:  ramification needed: Katz: 8/3
ExpPartsBarkatouReduction: Introduce the ramification c*x^3 for
system of size 4
ExpPartsBarkatouReduction:  apply Moser reduction for ramified
system
ExpPartsBarkatouReduction: splitting with ind. c time .457
ExpPartsBarkatouReduction: Smart ramification x^3
ExpPartsBarkatouReduction:  with splitting the system after
ramification time:  0.93e-1
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = t^3
1/x^11+(1/3)/x^7-(1/3)/x^3] with ramification x = t^3
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = t^3
-1/x^6] with ramification x = t^3
```

$$\left[x = t^3, \frac{1}{x^{11}} + \frac{1}{3\,x^7} - \frac{1}{3\,x^3}\right], \left[x = t^3, -\frac{1}{x^6}\right] \tag{1.1}$$

Exemple 2

$$> A := \frac{1}{x^7} \cdot \begin{bmatrix} x^5 & -x^3 & x^3-1 & -x^3 & -1 & -x^4 \\ 0 & 0 & x^2-x & x^6+x & x & -x \\ x^2 & x^6-x^5+x^2 & -x^3 & -x^3+1 & -x^4 & -x^5+x^3 \\ 0 & 0 & x-1 & x^6+x^4-x^3 & 1 & x^5 \\ 0 & x^2 & x^5 & -x^5 & -x^6+x^3 & -x^2 \\ -x^4 & x^6-x^4+x & -x^4 & 1 & -x^2 & x^2 \end{bmatrix} :$$

```
> ExpPartsBarkatouReduction(A, x, t)
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: Apply Moser
ExpPartsBarkatouReduction: The leading Matrix
ExpPartsBarkatouReduction: case 2: not nilpotente, for all roots
we split
ExpPartsBarkatouReduction:  with splitting the system time:
.592
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: Apply Moser
ExpPartsBarkatouReduction: The leading Matrix
ExpPartsBarkatouReduction: case 2: not nilpotente, for all roots
we split
ExpPartsBarkatouReduction:  with splitting the system time:
.253
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: Apply Moser
ExpPartsBarkatouReduction: The leading Matrix
ExpPartsBarkatouReduction: case 1: Nilpotente and moser
irreducible: compute Katz
ExpPartsBarkatouReduction: Time for computing Katz 0.8e-2
ExpPartsBarkatouReduction:  ramification needed: Katz: 9/2
ExpPartsBarkatouReduction: Introduce the ramification c*x^2 for
system of size 2
ExpPartsBarkatouReduction:  apply Moser reduction for ramified
system
ExpPartsBarkatouReduction: Smart ramification -2*x^2
ExpPartsBarkatouReduction:  with splitting the system after
ramification time:  0.93e-1
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = -2*
t^2 -(1/32)/x^11+(1/32)/x^10+(3/32)/x^9-(5/32)/x^8-(25/64)/x^7+
(41/16)/x^6-(261/8)/x^4+(13057/256)/x^3+(2879/8)/x^2] with
ramification x = -2*t^2
ExpPartsBarkatouReduction:  with splitting the system time:
1.001
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = t
RootOf(_Z^2+1)/x^6-RootOf(_Z^2+1)/x^5+1/x^5-2*RootOf(_Z^2+1)
/x^4+2/x^4-6*RootOf(_Z^2+1)/x^3+21/x^3+(55/2)*RootOf(_Z^2+1)
/x^2+131/x^2+(1129/2)*RootOf(_Z^2+1)/x+721/x] with ramification
x = t
ExpPartsBarkatouReduction:  with splitting the system time:
.448
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = t
RootOf(_Z^2+1)/x^7-(1/2)*RootOf(_Z^2+1)/x^6+(3/8)*RootOf(_Z^2+1)
/x^5+(1/2)/x^5-(5/16)*RootOf(_Z^2+1)/x^4-(173/128)*RootOf
(_Z^2+1)/x^3-(399/256)*RootOf(_Z^2+1)/x^2+(183/1024)*RootOf
```

```
(_Z^2+1)/x-3/x] with ramification x = t
```

$$\left[x = -2\,t^2, -\frac{1}{32\,x^{11}} + \frac{1}{32\,x^{10}} + \frac{3}{32\,x^9} - \frac{5}{32\,x^8} - \frac{25}{64\,x^7} + \frac{41}{16\,x^6} - \frac{261}{8\,x^4} + \frac{13057}{256\,x^3} + \frac{2879}{8\,x^2}\right], \left[x = t, \frac{RootOf\left(\_Z^2+1\right)}{x^6} - \frac{RootOf\left(\_Z^2+1\right)}{x^5} + \frac{1}{x^5} - \frac{2\,RootOf\left(\_Z^2+1\right)}{x^4} + \frac{2}{x^4} - \frac{6\,RootOf\left(\_Z^2+1\right)}{x^3} + \frac{21}{x^3} + \frac{55}{2}\,\frac{RootOf\left(\_Z^2+1\right)}{x^2} + \frac{131}{x^2} + \frac{1129}{2}\,\frac{RootOf\left(\_Z^2+1\right)}{x} + \frac{721}{x}\right], \left[x = t, \frac{RootOf\left(\_Z^2+1\right)}{x^7} - \frac{1}{2}\,\frac{RootOf\left(\_Z^2+1\right)}{x^6} + \frac{3}{8}\,\frac{RootOf\left(\_Z^2+1\right)}{x^5} + \frac{1}{2\,x^5} - \frac{5}{16}\,\frac{RootOf\left(\_Z^2+1\right)}{x^4} - \frac{173}{128}\,\frac{RootOf\left(\_Z^2+1\right)}{x^3} - \frac{399}{256}\,\frac{RootOf\left(\_Z^2+1\right)}{x^2} + \frac{183}{1024}\,\frac{RootOf\left(\_Z^2+1\right)}{x} - \frac{3}{x}\right] \quad (2.1)$$

Exemple 3

```
> A :=
```

$$\left[\left[\frac{4}{x^4}, 0, \frac{1}{x^7}, 0, 0, -\frac{5}{x^3} - \frac{1}{x}, 0, 0\right], \left[-\frac{5}{x^3}, -\frac{9}{x^2}, \frac{6}{x^5}, \frac{1}{x^7} - \frac{6}{x^6}, -\frac{2}{x^5}, 0, 0, 0\right], \left[0, \frac{1}{x^7}, -\frac{10}{x^5}, \frac{1}{x^3} - \frac{5}{x^2} - \frac{1}{x}, -\frac{2}{x^4}, \frac{10}{x^6}, 0, \frac{6}{x^2}\right], \left[0, 0, \frac{2}{x}, -\frac{1}{x^5}, 0, 0, -\frac{8}{x^4}, -\frac{5}{x^3}\right], \left[-\frac{7}{x^4}, -\frac{3}{x^3}, 0, -\frac{6}{x^5} - \frac{4}{x^3}, 0, -\frac{6}{x^4}, 0, \frac{8}{x^6}\right], \left[\frac{1}{x^7} - \frac{9}{x^2} - \frac{1}{x}, 0, 0, \frac{6}{x}, -\frac{10}{x^3}, \frac{1}{x^7} + \frac{8}{x^4}, 0, \frac{8}{x^4} - \frac{1}{x}\right], \left[\frac{2}{x}, 0, 0, \frac{1}{x^7} + \frac{1}{x^4}, 0, \frac{7}{x^3}, 0, -\frac{8}{x^5}\right], \left[-\frac{5}{x^2}, 0, 0, 0, 0, \frac{10}{x^5} - \frac{6}{x^3}, 0, -\frac{10}{x^6} + \frac{3}{x^3}\right]\right]:$$

```
> ExpPartsBarkatouReduction(A, x, t)
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: Apply Moser
ExpPartsBarkatouReduction: The leading Matrix
ExpPartsBarkatouReduction: case 2: not nilpotente, for all roots
```

```
we split
ExpPartsBarkatouReduction:  with splitting the system time:
.429
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = t
1/x^7+10/x^6-200/x^5+6968/x^4-298725/x^3+14242192/x^2
-725038151/x] with ramification x = t
ExpPartsBarkatouReduction:  with splitting the system time:
1.038
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: Apply Moser
ExpPartsBarkatouReduction: The leading Matrix
ExpPartsBarkatouReduction: case 1: Nilpotente and moser
irreducible: compute Katz
ExpPartsBarkatouReduction: Time for computing Katz 0.63e-1
ExpPartsBarkatouReduction:  ramification needed: Katz: 11/2
ExpPartsBarkatouReduction: Introduce the ramification c*x^2 for
system of size 7
ExpPartsBarkatouReduction:  apply Moser reduction for ramified
system
ExpPartsBarkatouReduction: splitting with ind. c time 3.462
ExpPartsBarkatouReduction: Smart ramification -10*x^2
ExpPartsBarkatouReduction:  with splitting the system after
ramification time:  0.78e-1
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = -10*
t^2 (1/1000000)/x^13-(1/200000)/x^12+(131/2000000)/x^11-
(19/20000)/x^10+(143239/8000000)/x^9-(69539/200000)/x^8+
(113459131/16000000)/x^7-(466768/3125)/x^6+
(82655665103/25600000)/x^5-(3560584523/50000)/x^4+
(408609419976357/256000000)/x^3-(1812600804291/50000)/x^2] with
ramification x = -10*t^2
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: Apply Moser
ExpPartsBarkatouReduction: The leading Matrix
ExpPartsBarkatouReduction: case 2: not nilpotente, for all roots
we split
ExpPartsBarkatouReduction:  with splitting the system time:
.840
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: Apply Moser
ExpPartsBarkatouReduction: The leading Matrix
ExpPartsBarkatouReduction: case 2: not nilpotente, for all roots
we split
ExpPartsBarkatouReduction:  with splitting the system time:
1.162
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = t^2
(1/2)*RootOf(_Z^2+32)/x^11-(1/6)/x^10-(29/864)*RootOf(_Z^2+32)
/x^9-(66137/31104)/x^8+(8799649/89579520)*RootOf(_Z^2+32)/x^7+
(556575917/806215680)/x^6+(27825936553/154793410560)*RootOf
(_Z^2+32)/x^5-(2519093693257/3482851737600)/x^4+
(9706680792390541/8024490403430400)*RootOf(_Z^2+32)/x^3-
(59329162490453969/108330620446310400)/x^2] with ramification x
= t^2
ExpPartsBarkatouReduction:  with splitting the system time:
1.204
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = t^2
(1/2)*RootOf(5*_Z^2-32)/x^11-(31/75)/x^10+(21137/86400)*RootOf
(5*_Z^2-32)/x^9+(171923293/97200000)/x^8-
(254308345693/559872000000)*RootOf(5*_Z^2-32)/x^7-
```

```
(161270312110441/62985600000000)/x^6+
(59660172017758747/120932352000000000)*RootOf(5*_Z^2-32)/x^5-
(1299003933848218 6199/1360488960000000000)/x^4+
(1421410099469277912836123/3134566563840000000000000)*RootOf(5*
_Z^2-32)/x^3-
(23952670363305410391926 4991/26447905382400000000000000)/x^2]
with ramification x = t^2
ExpPartsBarkatouReduction:  with splitting the system time:
.360
ExpPartsBarkatouReduction: a call of the algorithme
ExpPartsBarkatouReduction: we get an exponential part [x = t^2
-10/x^12+(4/25)/x^10-(17311/12500)/x^8+(687971/3125000)/x^6+
(12315211171/78125000)/x^4-(8445343634923 97/390625000000)/x^2]
with ramification x = t^2
```

$$\left[x = t, \frac{1}{x^7} + \frac{10}{x^6} - \frac{200}{x^5} + \frac{6968}{x^4} - \frac{298725}{x^3} + \frac{14242192}{x^2} - \frac{725038151}{x}\right], \left[x = \right. \tag{3.1}$$

$$-10\,t^2, \frac{1}{1000000\,x^{13}} - \frac{1}{200000\,x^{12}} + \frac{131}{2000000\,x^{11}} - \frac{19}{20000\,x^{10}} + \frac{143239}{8000000\,x^9}$$

$$- \frac{69539}{200000\,x^8} + \frac{113459131}{16000000\,x^7} - \frac{466768}{3125\,x^6} + \frac{82655665103}{25600000\,x^5} - \frac{3560584523}{50000\,x^4}$$

$$+ \frac{408609419976357}{256000000\,x^3} - \frac{1812600804291}{50000\,x^2}\Bigg], \left[x = t^2, \frac{1}{2}\,\frac{RootOf\left(\_Z^2 + 32\right)}{x^{11}}\right.$$

$$- \frac{1}{6\,x^{10}} - \frac{29}{864}\,\frac{RootOf\left(\_Z^2 + 32\right)}{x^9} - \frac{66137}{31104\,x^8}$$

$$+ \frac{8799649}{89579520}\,\frac{RootOf\left(\_Z^2 + 32\right)}{x^7} + \frac{556575917}{806215680\,x^6}$$

$$+ \frac{27825936553}{154793410560}\,\frac{RootOf\left(\_Z^2 + 32\right)}{x^5} - \frac{2519093693257}{3482851737600\,x^4}$$

$$+ \frac{9706680792390541}{8024490403430400}\,\frac{RootOf\left(\_Z^2 + 32\right)}{x^3} - \frac{59329162490453969}{108330620446310400\,x^2}\Bigg], \left[x = t^2,\right.$$

$$\frac{1}{2}\,\frac{RootOf\left(5\,\_Z^2 - 32\right)}{x^{11}} - \frac{31}{75\,x^{10}} + \frac{21137}{86400}\,\frac{RootOf\left(5\,\_Z^2 - 32\right)}{x^9}$$

$$+ \frac{171923293}{97200000\,x^8} - \frac{254308345693}{559872000000}\,\frac{RootOf\left(5\,\_Z^2 - 32\right)}{x^7} - \frac{161270312110441}{62985600000000\,x^6}$$

$$+ \frac{59660172017758747}{120932352000000000}\,\frac{RootOf\left(5\,\_Z^2 - 32\right)}{x^5} - \frac{12990039338482186199}{1360488960000000000\,x^4}$$

$$+ \frac{1421410099469277912836123}{3134566563840000000000000}\,\frac{RootOf\left(5\,\_Z^2 - 32\right)}{x^3}$$

$$- \frac{2395267036330541039192649 91}{26447905382400000000000000\,x^2}\Bigg], \left[x = t^2, -\frac{10}{x^{12}} + \frac{4}{25\,x^{10}} - \frac{17311}{12500\,x^8}\right.$$

$$\left\lfloor \quad + \frac{687971}{3125000\,x^6} + \frac{12315211171}{78125000\,x^4} - \frac{844534363492397}{390625000000\,x^2} \right\rfloor$$